China Ocean Engineering, Vol. 13, No. 4, pp.429 – 442.
China Ocean Press, 1999. Printed in P. R. China.

Breaking of Solitary Waves on Uniform Slopes

CHOU Chung-Ren(周宗仁)* and OUYANG Kwan(欧阳宽)**

Abstract — The propagation, shoaling and breaking of solitary waves on mild slopes are simulated by boundary element method. In this paper, the criterion of breaking solitary waves on mild slopes is discussed. The criterion is that the ratio of horizontal velocity of water particles on the wave crest to wave celerity equals one. However, the case that the ratio of horizontal velocity of water particles on the wave crest to wave celerity is below one but the front face of wave profile becomes vertical is also considered as a breaking criterion. According to the above criteria, the breaking index for slopes 1:10 to 1:25 is studied. The result is compared to other researchers'. The deformation of solitary waves on slopes is discussed and the distribution of fluid velocities at breaking is shown.

Key words: *breaking wave*; *solitary wave*; *distribution of fluid velocity*; *boundary element*

1. Introduction

Usually large amounts of energy released by wave breaking will cause serious damage to coastal structures, thus, it is important to realize the characteristics of shoaling and breaking, including the deformation of wave profile, the location and height of wave breaking. Earlier researchers (Ippen, 1966) obtained experimental results for shoaling and breaking of solitary waves. Based on laboratory experiments, the types of breaker are classified (Street and Camfield, 1966). For a solitary wave passing through a channel of a constant width, or of a linearly decreasing width, Seaki *et al*. (1971) discussed the changes in wave height, breaking point, and the height of run-up. Using an approximate theory, Synolakis (1987) derived the maximum run-up for solitary waves. By use of numerical simulation, Madsen and Mei (1969) simulated the deformation of solitary waves passing through a mild slope onto a shelf. Nakayama (1983) analyzed the propagation of tsunami and run-up of a solitary wave against a vertical wall. Based on the mixed Eulerian-Lagrangian technique, Kioka (1983) presented the deformation and velocity field for plunging and spilling breakers. Using the boundary integral equation solution, Kim *et al*. (1983) discussed the generation, propagation and run-up of both a solitary wave and two successive solitary waves. Based on the long-wave equation with curvature effects, Seabra-Santos *et al*. (1987) studied the evolution of a solitary wave near an obstacle or over an uneven bottom. Grilli *et al*. investigated the breaking of solitary waves on slopes (1994a; 1997) and on a submerged breakwater (1994b). Using boundary element, Chou and Shih(1996) simulated the generation and deformation of solitary waves propagating on slopes.

To determine whether waves break or not, many researchers are interested in breaking criteria. Stokes (1883) proposed a limiting wave profile and revealed that its critical angle is 120° in constant water depth. Longuet-Higgins (1982) used the elementary function to describe breaking waves and proved again the limiting wave profile proposed by Stokes. Another type of breaking criterion is the

* Professor, Department of Harbor and River Engineering, Taiwan Ocean University, Keelung, China
** Graduate Student, Department of Harbor and River Engineering, Taiwan Ocean Unversity, Keelung, China

related breaking wave height $H_b' = H_b / h_b$, where H_b is the breaking wave height and h_b is the local water depth at breaking. Different researchers have given different values of H_b', for example, $H_b' = 0.73$ (Boussinesq, 1891), $H_b' = 0.78$ (McCown, 1894), $H_b' = 0.83$ (Yamada, 1968) and $H_b' = 0.75$ (Kishi, 1964).

The breaking criterion mentioned above is valid only for constant water depth. For waves propagating on slopes, Camfield and Street (1969) proposed an empirical breaking criterion of related breaking height as a function of slope. Based on shallow water equation, Dean and Dalrymple (1984) derived the location of breaking. Using the fully nonlinear potential flow wave model, Grilli *et al*. (1997) presented the criterion of breaking type and breaking index.

To realize the characteristics of breaking waves, solitary waves with various incident wave heights on different slopes are simulated. In this numerical model, the fully nonlinear free water surface boundary condition is used. Linear elements are adopted and the double-node technique is used to solve the corner problem. The deformation of waves on slopes is presented and the distribution of fluid velocities at breaking is shown. Breaking indices for slopes 1:10 to 1:25 are summarized.

2. Theoretical Analysis

2.1 Governing Equation

The two-dimensional boundary element method with fully nonlinear boundary condition on free water surface is used to simulate solitary waves propagating on mild slopes. The fluid is assumed to be inviscid and incompressible, and the flow is irrotational. The governing equation can be expressed in terms of velocity potential $\Phi(x, z; t)$ as

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{1}$$

where x and z are spatial coordinates in horizontal and vertical directions as shown in Fig. 1. The symbols of Γ_1, Γ_2, Γ_3 and Γ_4 in Fig. 1 denote the boundaries of pseudo moving wave generator, free water surface, impermeable slope and seabed, respectively, and h_0 is the constant depth of water.

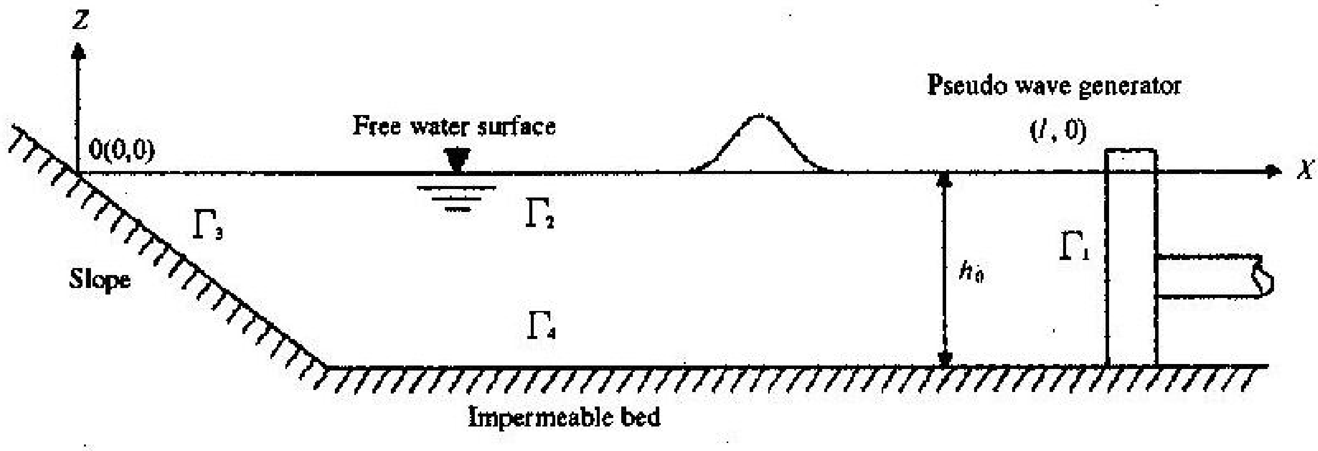

Fig. 1. Definition of numerical wave tank.

2.2 Integral Equation

According to Green's Second Identity, if the velocity potential satisfies the Laplace equation and its second derivative exists, the velocity potential can be obtained by the velocity potential on the boundary, $\Phi(\xi, \eta; t)$, and its normal derivative, $\partial\Phi(\xi,\eta;t)/\partial n$, i.e.

$$c\Phi(x, z; t) = \frac{1}{2\pi}\int_{\Gamma}\left[\frac{\partial\Phi(\xi, \eta; t)}{\partial n}\ln\frac{1}{r} - \Phi(\xi,\eta;t)\frac{\partial}{\partial n}\ln\frac{1}{r}\right] ds$$

$$c = \begin{cases} 1 & \text{inside fluid domain} \\ 1/2 & \text{on smooth boundary} \\ 0 & \text{outside fluid domain} \end{cases} \tag{2}$$

where $r = [(\xi - x)^2 + (\eta - z)^2]^{1/2}$.

2.3 Boundary Conditions

For a piston-type wave generator, the fluid velocity normal to the paddle should be the same as the horizontal moving velocity of wave generator $U(t)$:

$$\frac{\partial\Phi}{\partial n} = -U(t), \qquad \text{on } \Gamma_1 \tag{3}$$

where n denotes the unit outward normal vector. For generation of a solitary wave, $U(t)$ in Eq. (4) can be expressed as

$$U(t) = H_0\sqrt{\frac{g}{h}}\operatorname{sech}^2\left[\sqrt{\frac{3H_0}{4h^3}}\, C\,(t - t_c)\right] \tag{4}$$

where H_0, g and h are incident wave height, gravitational acceleration and water depth, respectively, C denotes the wave celerity of solitary wave and is obtained by $C = \sqrt{g(H + h)}$, and t_c represents the characteristic time, which is half time of the movement of paddle. For the boundary condition on free water surface, fully nonlinear boundary condition is applied, with the assumption that the atmospheric pressure is constant and equal to zero:

$$\frac{D\Phi}{Dt} + g\eta - \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right] = 0 \tag{5}$$

where D denotes the Lagrangian differentiation and η is the elevation of free water surface. On an impermeable slope and seabed, the velocities normal to boundaries are equal to zero.

2.4 Numerical Method

To solve Eq. (2) numerically, linear elements are adopted for all boundaries, say, Γ_1 to Γ_4.

The double-node technique (Chou, 1988) is applied to the corners which have two different boundary conditions. Based on the Lagrangian description, the following relation is obtained:

$$u = \frac{Dx}{Dt} = \frac{\partial \Phi}{\partial x}; \tag{6}$$

$$w = \frac{Dz}{Dt} = \frac{\partial \Phi}{\partial z}. \tag{7}$$

The forward-difference in time is used to obtain the position of free water surface at the next time. This difference technique is also adopted in Eq. (5) to obtain updated velocity potential. The horizontal and vertical components of fluid velocity in fluid field are

$$u = \frac{\partial \Phi(x, z; t)}{\partial x} = \frac{1}{2\pi}\int_\Gamma \left\{ \frac{\partial \Phi(\xi, \eta; t)}{\partial n}\left(\frac{\xi - x}{r^2}\right) - \Phi(\xi, \eta; t)\int_\Gamma \left[\frac{\partial x}{\partial n}\left(\frac{1}{r^2} - \frac{2(\xi - x)^2}{r^4}\right) - \frac{\partial z}{\partial n}\left(\frac{2(\xi - x)(\eta - z)}{r^4}\right)\right]\right\} \mathrm{d}\Gamma ; \tag{8}$$

$$w = \frac{\partial \Phi(x, z; t)}{\partial z} = \frac{1}{2\pi}\int_\Gamma \left\{ \frac{\partial \Phi(\xi, \eta; t)}{\partial n}\left(\frac{n - z}{r^2}\right) - \Phi(\xi, \eta; t)\int_\Gamma \left[\frac{\partial z}{\partial n}\left(\frac{1}{r^2} - \frac{2(\eta - z)^2}{r^4}\right) - \frac{\partial z}{\partial n}\left(\frac{2(\xi - x)(\eta - z)}{r^4}\right)\right]\right\} \mathrm{d}\Gamma . \tag{9}$$

About the details can be referred to Chou *et al*.

3. Numerical Results and Discussion

3.1 Breaking Criteria for Solitary Waves on Slopes

Many breaking criteria have been studied. For constant water depth, Stokes (1883), proved that when the ratio of horizontal velocity of water particle on wave crest u to wave celerity C equals one, the limiting angle of wave profile on crest equals 120 °. The related breaking wave height H_b/h_b is usually adopted, in this paper $H_b/h_b = 0.78$ derived by McCown (1894) is used. Grilli *et al*. (1997) defined the breaking criterion of solitary waves on slopes when the front face of wave profile has a vertical tangent.

For the understanding of the characteristics of waves which reach the above criteria and determination of a suitable criterion for wave breaking on slopes, waves with height $H_0/h_0 = 0.1$ to 0.4 propagating on slopes $s = 1:10$ to $1:25$ are studied. The wave characteristics for each criterion are summarized in Tables 1 to 4. The horizontal fluid velocity u on wave crest is calculated by averaging three nodes near wave crest. From the results of simulation, it is found that these criteria of $H/h = 0.78$ occur first, then the angle of wave crest $\theta = 120°$, $u/C = 1$ occur, finally the front face of wave becomes vertical. First of all, $H_0/h_0 = 0.3$ and $s = 1:15$ are analyzed to determine which criterion is suitable for breaking criteria.

In Table 1, the characteristics of waves which reach the criterion $H/h = 0.78$ are listed. It indicates that the angle of crest is 165 ° and u/C is 0.53. Fig. 2 shows the time histories of fluid mass, total energy, the horizontal velocity of water particle on wave crest u and wave celerity C. The criterion $H/h = 0.78$ occurs at $t = 5.5t_c$, and the conservation of mass and energy is kept. For this moment, the wave profile is shown in Fig. 3 (curve a), and slight asymmetry appears. Fig. 4(a) shows the uniform distribution of fluid velocity. There is no scene that wave will break.

The criterion $\theta = 120°$ is reached at $t = 5.7t_c$. From Table 2, $u/C = 0.93$ and $H/h = 1.74$. The fluid mass and energy keep conservative, the wave profile (curve b in Fig.3) becomes asymmetric and the wave height increases. The fluid velocities on the front face of wave profile increase but the wave does not break.

Table 1 The location of wave crest, water depth, wave height, angle of wave profile and u/C at $H/h = 0.78$.

Slope	1:10				1:15			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
x/h_0		2.87	4.08	5.58	2.44	4.76	6.69	8.61
h/h_0		0.29	0.41	0.56	0.16	0.32	0.45	0.57
H/h_0		0.20	0.34	0.45	0.13	0.25	0.35	0.46
θ		170°	160°	156°	167°	163°	165°	155°
u/C		0.53	0.52	0.52	0.47	0.51	0.53	0.53
Slope	1:10				1:15			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
x/h_0		6.59	9.42	11.97	4.60	8.82	12.29	15.19
h/h_0		0.33	0.47	0.60	0.18	0.35	0.49	0.61
H/h_0		0.26	0.37	0.47	0.15	0.28	0.38	0.48
θ		159°	151°	151°	162°	156°	152°	148°
u/C		0.53	0.54	0.53	0.51	0.54	0.51	0.53

Table 2 The location of wave crest, related wave height and u/C at $\theta = 120°$.

Slope	1:10				1:15			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
x/h_0			1.54	2.19		2.65	3.52	3.81
H/h			2.36	1.42		1.57	1.74	1.58
u/C			0.86	0.74		0.79	0.93	0.93
Slope	1:20				1:25			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
x/h_0	3.86	4.46	6.16	8.20		6.40	9.00	13.12
H/h	0.75	1.24	1.40	1.26		1.32	1.26	0.91
u/C	0.55	0.80	0.87	0.83		0.83	0.84	0.70

The criterion $u/C = 1$ is reached at $t = 5.745t_c$. From Table 3, $\theta = 113°$ and $H/h = 1.75$. At this moment, the front face of wave profile (curve c in Fig.3) becomes evidently steep and the fluid velocities on crest become nonuniform. Because of $u/C = 1$ the water particles will deviate from the wave profile, thus a breaking criterion is considered. For the criterion that the front face of wave profile has a vertical tangent, the wave characteristics are summarized in Table 4. For some cases, e.g., for waves with incident height $H_0/h_0 = 0.2$ propagating on slope $s = 1:15$, when the front face of waves becomes vertical, the instability of wave profile is considered as a breaking criterion even if u/C is smaller than one.

Table 3 The location of wave crest, angle of wave profile and related wave height at $u/C = 1$.

Slope	1:10				1:15			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
u/C	0.25*	0.67*	1.00	0.76*	0.51*	0.92*	1.00	1.00
x/h_0	3.96	1.57	1.54	3.02	2.08	2.45	3.31	4.43
θ	174°	144°	116°	83°	110°	97°	113°	108°
H/h_0	0.29	1.56	2.36	1.42	0.87	1.57	1.75	1.76
Slope	1:20				1:25			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
u/C	0.55*	1.00	1.00	0.90*	1.00	1.00	1.00	1.00
x/h_0	3.77	4.48	5.86	7.59	2.67	6.08	8.45	12.65
θ	102°	132°	122°	80°	147°	103°	106°	106°
H/h	0.65	1.22	1.42	1.29	1.68	1.58	1.44	0.90

Table 4 The location of wave crest, related wave height, angle of wave profile and u/C when the front face of wave profile has a vertical tangent.

Slope	1:10				1:15			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
x/h_0			1.48	3.02		2.45	3.28	4.27
H/h_0			0.375	0.428	2	0.258	0.391	0.517
H/h			2.54	1.42		1.57	1.79	1.81
θ			84°	83°		87°	86°	81°
u/C			0.99	0.76		0.92	1.07	1.06
Slope	1:20				1:25			
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
x/h_0	3.77	4.32	6.16	7.59	2.73	5.96	8.26	12.64
H/h_0	0.123	0.278	0.44	0.511	0.166	0.352	0.458	0.467
H/h	0.65	1.29	1.56	1.29	1.53	1.48	1.39	0.92
θ	102°	88°	86°	80°	83°	85°	82°	81°
u/C	0.55	1.71	1.78	0.90	1.62	2.68	1.12	1.76

3.2 Breaking Index

The breaking criterion is defined as when the ratio of horizontal velocity of water particle on wave crest to wave celerity equals one. However, the case that the ratio of the horizontal velocity of water particle on wave crest to the wave celerity is below one but the front face of wave profile has a vertical tangent is also considered as a breaking criterion. According to the above criteria, breaking indices for slopes $s = 1:10$ to $1:25$ are summarized in Table 5. Breaking indices derived by Grilli *et al*. (1997) are also listed. For slope $s = 1:10$, the related breaking heights obtained by Grilli *et al*. seem to be on the large side. For waves with H_0/h_0 larger than 0.3 propagating on slopes milder

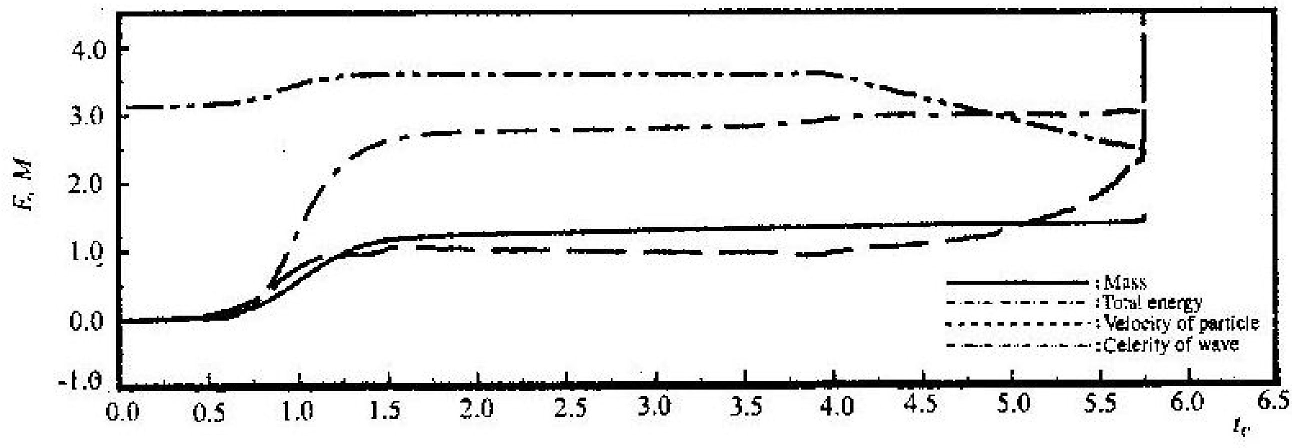

Fig. 2. Time histories of mass, total energy, velocity of particle and celerity for $H_0/h_0 = 0.3$ on slope 1:15.

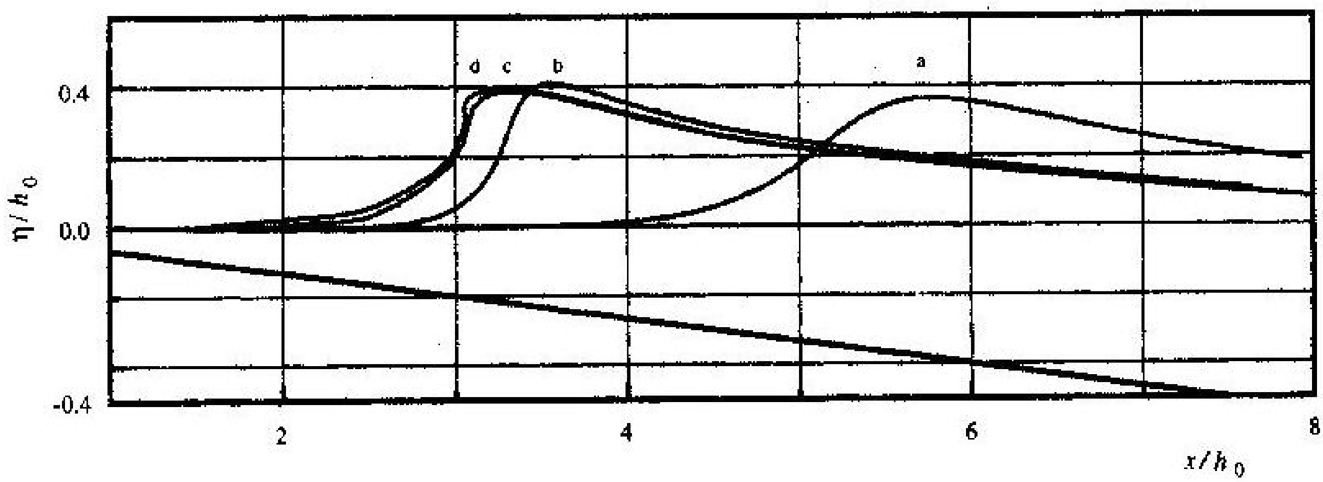

Fig. 3. Deformation for $H_0/h_0 = 0.3$ on slope 1:15 (curve a: $H/h = 0.78$, curve b: $\theta = 120°$, cuve c: $u/C = 1$, curve d: a vertical tangent occurs on the front face of wave profile).

than $s = 1:20$, good agreements are obtained for breaking height H_b/h_b, but Grilli *et al*.'s related breaking heights H_b/h_0 are larger than the present results. Grilli *et al*.'s locations of breaking point are nearer to the shoreline than the present results. For other cases, the breaking heights H_b/h_0 derived by Grilli *et al*. are larger than the authors'. This is because in the authors' numerical scheme, linear elements are used, and the five-point smoothing technique is adopted after the breaking criterion is reached, therefore only the initial shape of breaker jet can be formed; in Grilli *et al*.'s scheme, the breaker jet of plunging breaker was performed successfully by using the quasi-singular integration and regriding technique. With this numerical technique the locations of breaking points are too close to the shoreline and the breaking wave heights are overestimated.

From Table 4, it can be seen that when the front face of wave profile has a vertical tangent and the angle of wave profile on crest is smaller than 90 ° and it will be classified as plunging breaker. For other cases, the breaker is classified as surging breaker.

3.3 Deformation of Wave Profile and Distribution of Fluid Velocities

Fig. 5 shows the deformation of solitary waves on slope $s = 1:10$. When a wave with $H_0/h_0 = 0.1$ (Fig. 5 (a)) approaches the shoreline, bore occurs. For $H_0/h_0 = 0.2$ (Fig.5 (b)), the front face of wave profile becomes tilted forward to the shoreline at breaking. For $H_0/h_0 = 0.3$ and 0.4 (Figs. 5(c) and 5(d)), plunging breakers are formed.

Fig. 6 to Fig. 8 show the deformation of solitary waves on slopes $s = 1:15$ to $1:25$; the results are the same as in Fig. 5.

Table 5 Breaking indices for solitary waves on slopes $s = 1:10$ to $1:25$

Slope	1:10								1:15							
H_0/h_0	0.1		0.2		0.3		0.4		0.1		0.2		0.3		0.4	
H_0/h_0	0.11		0.24		0.36	0.77*	0.43	0.77*	0.12		0.26	0.51*	0.39	0.53*	0.52	0.56*
H_b/h_0	0.29		1.56		2.36	4.98*	1.42	3.94*	0.87		1.58	3.61*	1.76	2.76*	1.77	2.35*
h_b/h_0	0.40		0.16		0.15	0.16*	0.30	0.19*	0.14		0.16	0.14*	0.22	0.19*	0.30	0.24*
x_b/h_0	3.95		1.56		1.54	1.55*	3.02	1.94*	2.13		2.45	2.09*	3.31	2.88*	4.43	3.60*
u/C	0.25		0.67		1.00		0.76		0.51		0.92		1.00		1.00	
θ	174°		144°		116°		83°		147°		87°		113°		108°	
Type of breaker	Surge	Surge*	Surge	Surge*	Plunge	Plunge*	Plunge	Plunge*	Surge	Surge*	Plunge	Plunge*	Plunge	Plunge*	Plunge	Plunge*
Slope	1:10								1:15							
H_0/h_0	0.1		0.2		0.3		0.4		0.1		0.2		0.3		0.4	
H_0/h_0	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4								
H_0/h_0	0.12	0.37*	0.27	0.41*	0.42	0.46*	0.51	0.51*	0.18	0.31*	0.38	0.37*	0.49	0.43*	0.47	0.49*
H_b/h_b	0.65	3.94*	1.22	2.51*	1.42	2.05*	1.29	1.82*	1.67	2.89*	1.58	2.01*	1.44	1.72*	0.92	1.56*
h_b/h_0	0.19	0.09*	0.22	0.16*	0.29	0.22*	0.40	0.28*	0.11	0.11*	0.24	0.18*	0.34	0.25*	0.51	0.31*
x_b/h_0	3.76	1.88	4.48	3.24*	5.86	4.46*	7.95	5.59*	2.67	2.65*	6.08	4.56*	8.45	6.26*	12.65	7.85*
u/C	0.55		1.00		1.00		0.90		1.00		1.00		1.00		1.00	
θ	102°		132°		122°		83°		147°		103°		106°		106°	
Type of breaker	Surge	Plunge*	Plunge	Plunge*	Plunge	Plunge*	Plunge	Plunge*	Plunge	Plunge*	Plunge	Plunge*	Plunge	Plunge*	Plunge	Plunge*

* — breaking indices derived by Grilli *et al*.

Fig. 9 presents the distribution of fluid velocities on slope $s = 1:20$ at breaking; strong velocities on the front of wave crest are observed.

4. Conclusion

Solitary waves with incident heights $H_0/h_0 = 0.1$ to 0.4 propagating on slopes $s = 1:10 \sim 1:25$ are studied. Breaking criterion is defined when the ratio of horizontal velocity of water particle on wave crest to wave celerity equals one. In addition, if the ratio of horizontal velocity of water particle on wave crest to wave celerity is smaller than one but the front face of wave profile become vertical, it is also considered as a breaking criterion. According to the above criteria, breaking indices are summarized. The breaker type is classified by the angle of wave profile on crest at breaking. If it is smaller than 90°, the wave will be considered as a plunging breaker, otherwise it will be classified into surging breaker.

References

Boussinesq, J., 1891. Essai Sur La Theorie Des eaux Courantes, Institute de France Academes des Science Mem divers

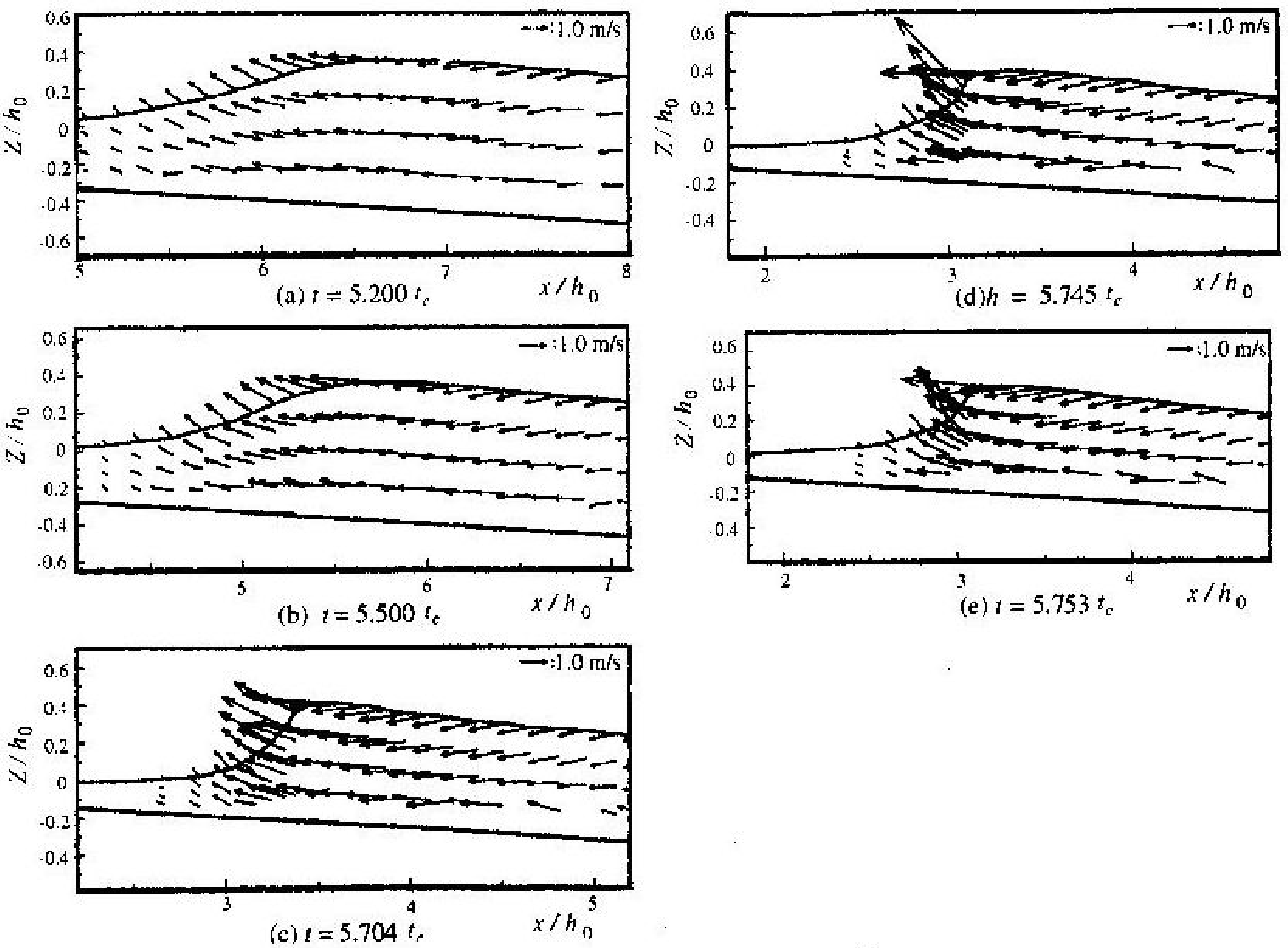

Fig. 4. Distributions of fluid velocities for $H_0/h_0 = 0.3$ on slope $s = 1:15$.

savants, **32**. (in French)

Camfield, F. E. and R. L. Street, 1969. Shoaling of Solitary Waves on Small Slopes, *J. of Waterway, Port, Coastal and Ocean Eng.*, ASCE, **95**(1), 1 ~ 22.

Chou, C. R., 1983. *The Boundary Element Method Applied to Waves*, Dept. of Harbor and River Eng. of Taiwan Ocean University, Keelung. (in Chinese)

Chou, C. R. and R. S. Shih, 1996. Generation and Deformation of Solitary Waves, *China Ocean Engineering*, **10** (4), 419 ~ 432.

Chou, C. R., C. Y. Yueh and Weng, W. K., 1988. Numerical Solution for Nonlinear Wave, *Proc. 10th Conf. On Ocean Eng., Taiwan*, 617 ~ 627. (in Chinese)

Dean, R. G. and R. A. Dalrymple, 1984. *Water Wave Mechanics for Engineers and Scientists*, Prentice-Hall, New Jersey, U.S.A.

Grilli, T. S. and R. Subramanya, 1994a. Quasi-singular Integrals in the Modeling of Nonlinear Water Waves in Shallow Water, *Engrg. Anal. with Boundary Elements*, **6**(2), 97 ~ 107.

Grilli, T. S., M. A. Losada and F. Martin, 1994b. Characteristics of Solitary Wave Breaking Induced by Breakwater, *J. of Waterway, Port, Coastal and Ocean Eng.*, **120**, 74 ~ 92.

Grilli, S. T., I. A. Svendsen and R. Subramaya, 1997. Breaking Criterion and Characteristics for Solitary Waves on Slopes, *J. of Waterway, Port, Coastal and Ocean Eng.*, **123**(3), 120 ~ 112.

Ippen, T., 1966. *Estuary and Coastline Hydrodynamics*, MaGraw-Hill Inc., **24**.

Kim, S. K., P. L-F. Liu and J. A. Liggett, 1983. Boundary Integral Equation Solutions for Solitary Wave Generation, Propagation and Run-up, *Coastal Engineering*, **7**, 299 ~ 311.

Kioka, W., 1983. Numerical Analysis of Breaking Waves in a Shallow Water, *Coastal Engineering in Japan*, **26**, 11 ~ 18.

Kishi, T. and Saeki, H., 1964. A Study of Cnoidal Wave, *Proceeding of Coastal Eng. in Japan*, **11**, 7 ~ 11.

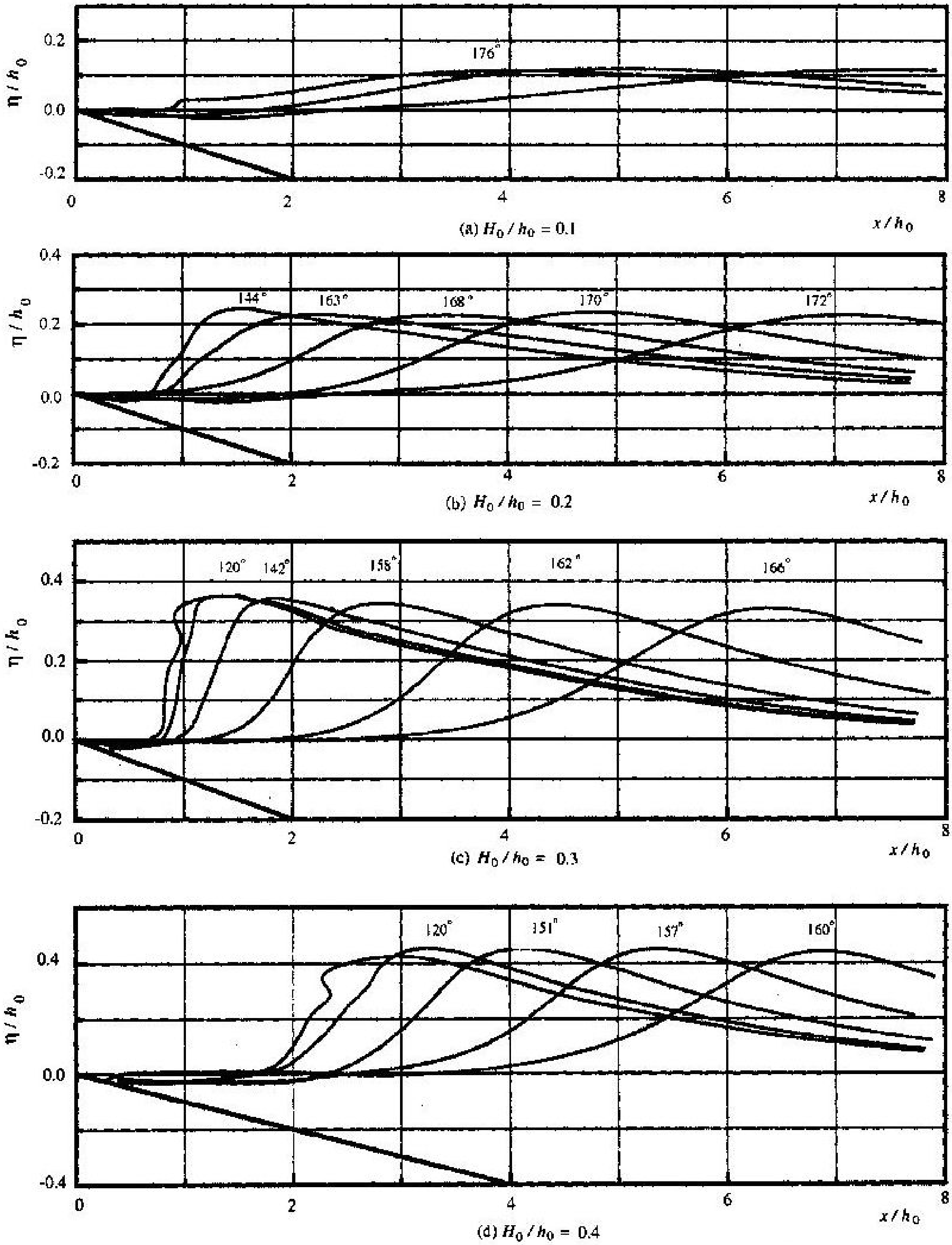

Fig. 5. Deformation of solitary waves on slope $s = 1:10$.

Longuet-Higgins, M. S., 1982. Parametric Solution for Breaking Waves, *J. Fluid Mech.*, **121**, 403 ~ 424.

Madsen, O. S. And C. C. Mei, 1969. The Transformation of a Solitary Wave over an Uneven Bottom, *J. Fluid Mech.*, **39**(part 4), 781 ~ 791.

McCowan, J., 1894. On the Highest Wave of Permanent Type, *Phil. Mag.*, **38**(5), 351 ~ 357.

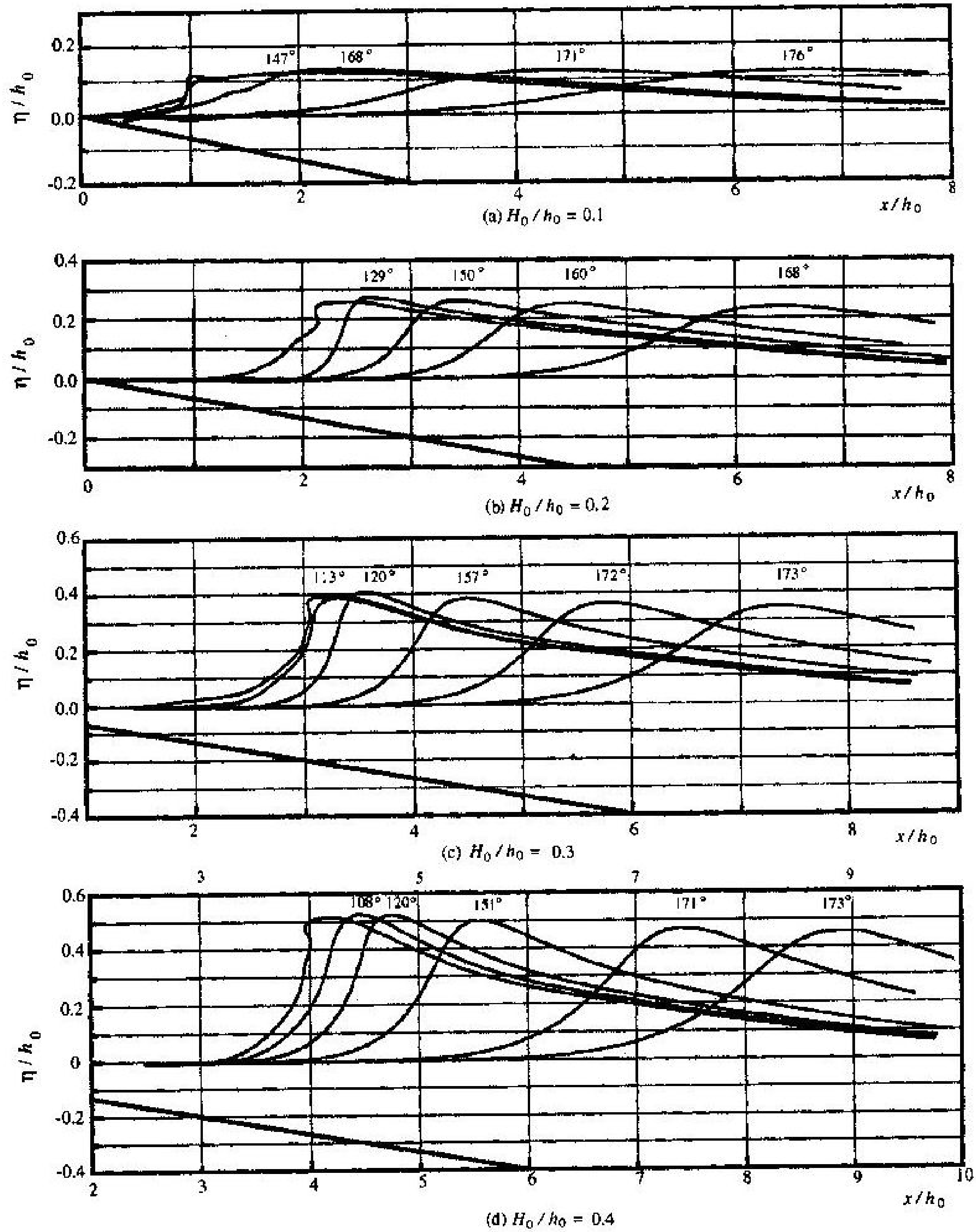

Fig. 6. Deformation of solitary waves on slope $s = 1:15$.

Nakayama, T., 1983. Boundary Element Analysis of Nonlinear Water Wave Problems, *International J. for Numerical Method in Eng.*, **19**, 953 ~ 970.

Otta, A. K., I. A. Svendsen and I. A. Grilli, 1993. The Breaking and Runup of Solitary Waves on Water, *Proc.*, 23*rd Int. Conf. on Coast. Eng.*, **2**, ASCE, New York, N. K., 1461 ~ 1474.

Seabra-santos, F. J., D. P. Renouard and A. M. Temperville, 1987. Numerical and Experimental Study of the Transformation of a Solitary Wave over a Shelf or Isolated Obstacle, *J. Fluid Mech.*, **176**, 117 ~ 134.

Seaki, H., S. Hanayasu, K. Ozaki and K. Takagi, 1971. The Shoaling and Run-up Height of the Solitary Wave,

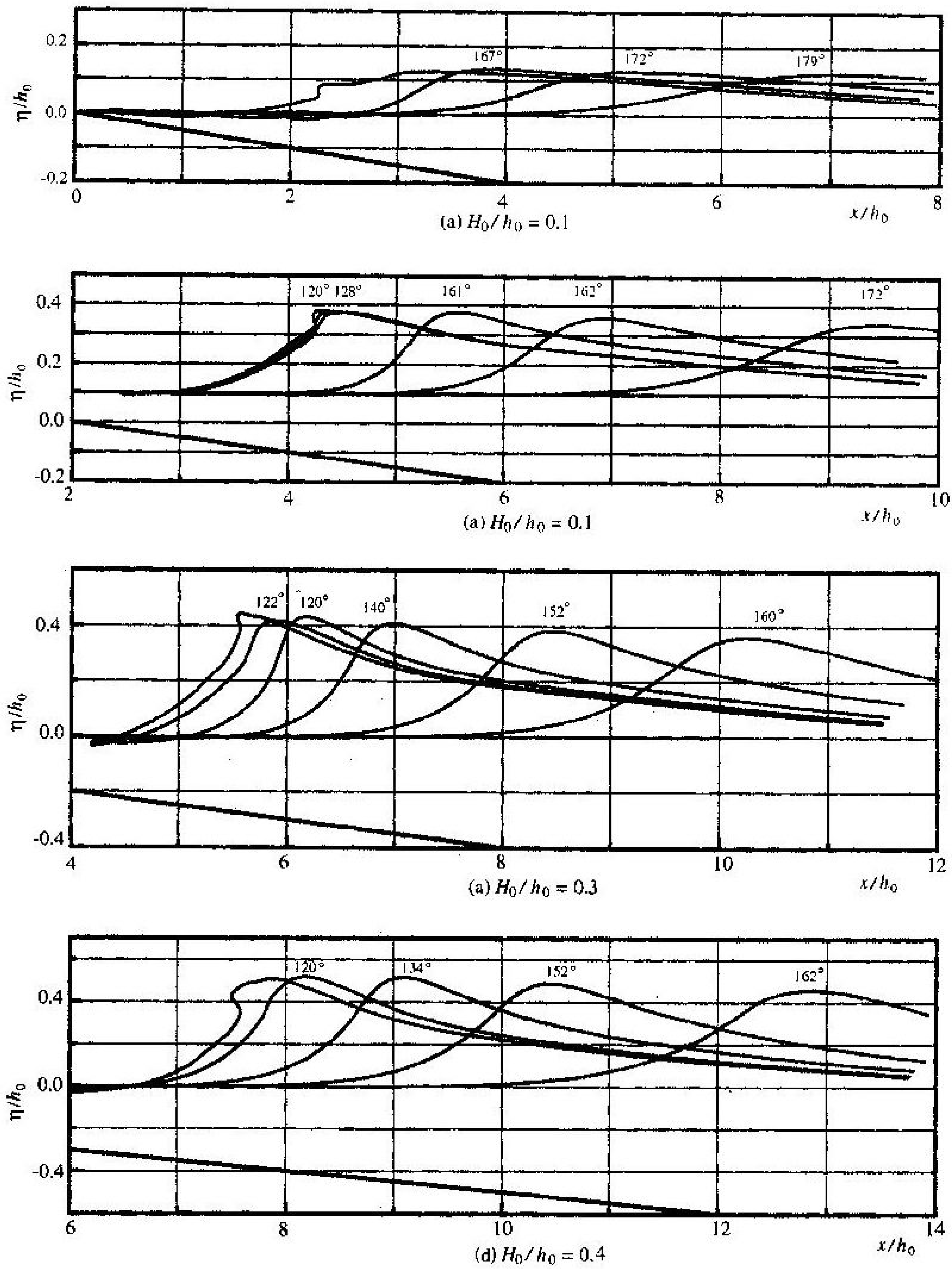

Fig. 7. Deformation of solitary waves on slope $s = 1:20$.

Coastal Engineering in Japan, **14**, 25 ~ 42.

Stokes, G. G., 1883. On the Highest Waves of Uniform Propagation, *Proceedings Camb. Phil.*, Sec. 4, 361 ~ 365.

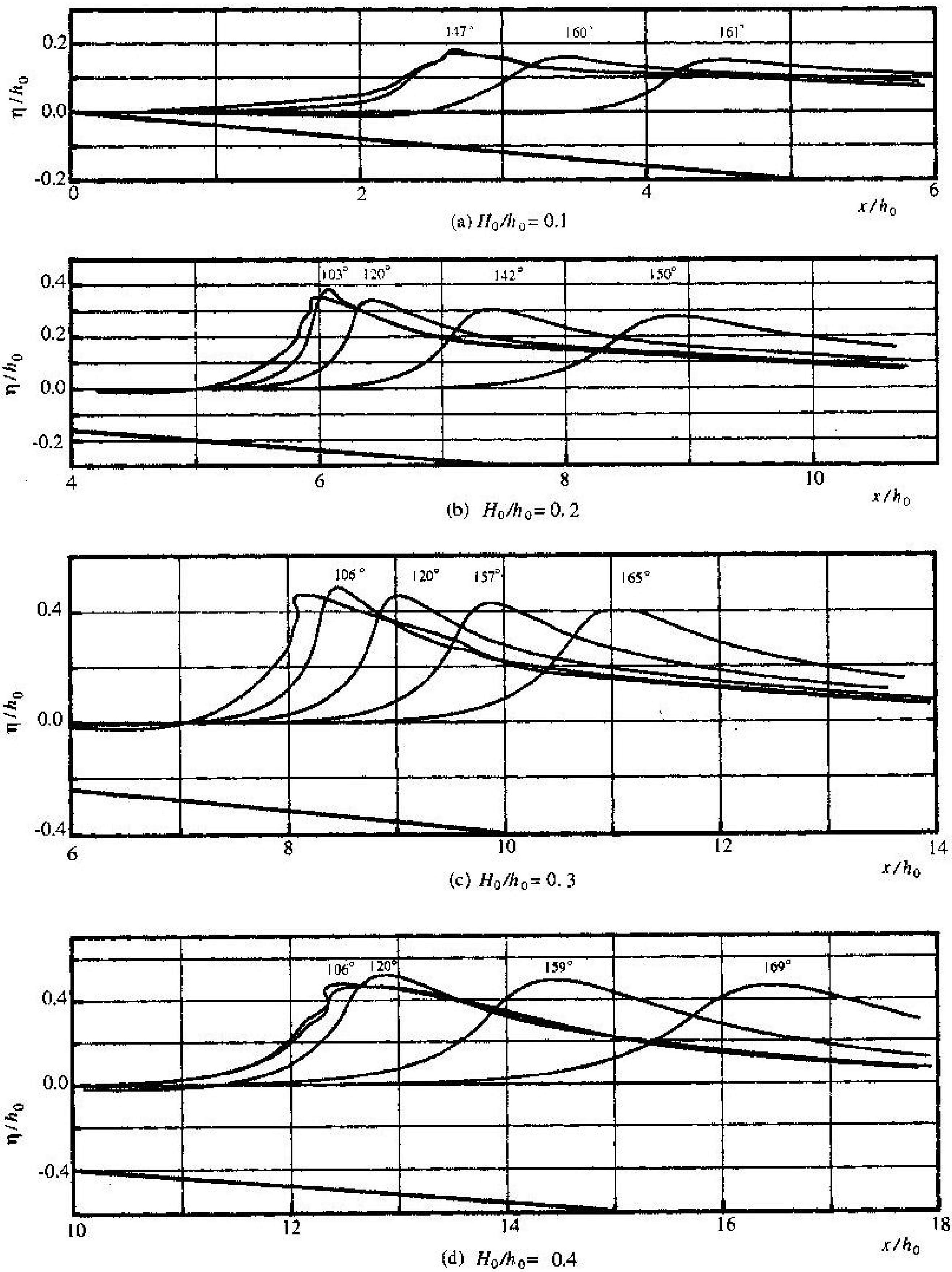

Fig. 8. Deformation of solitary waves on slope s = 1:25 .

Street, R. L. and F. E. Camfield, 1966. Observations and Experiments on Solitary Wave Deformation, *Proc. 10th Conf. Coastal Eng.*, 284 ~ 301.

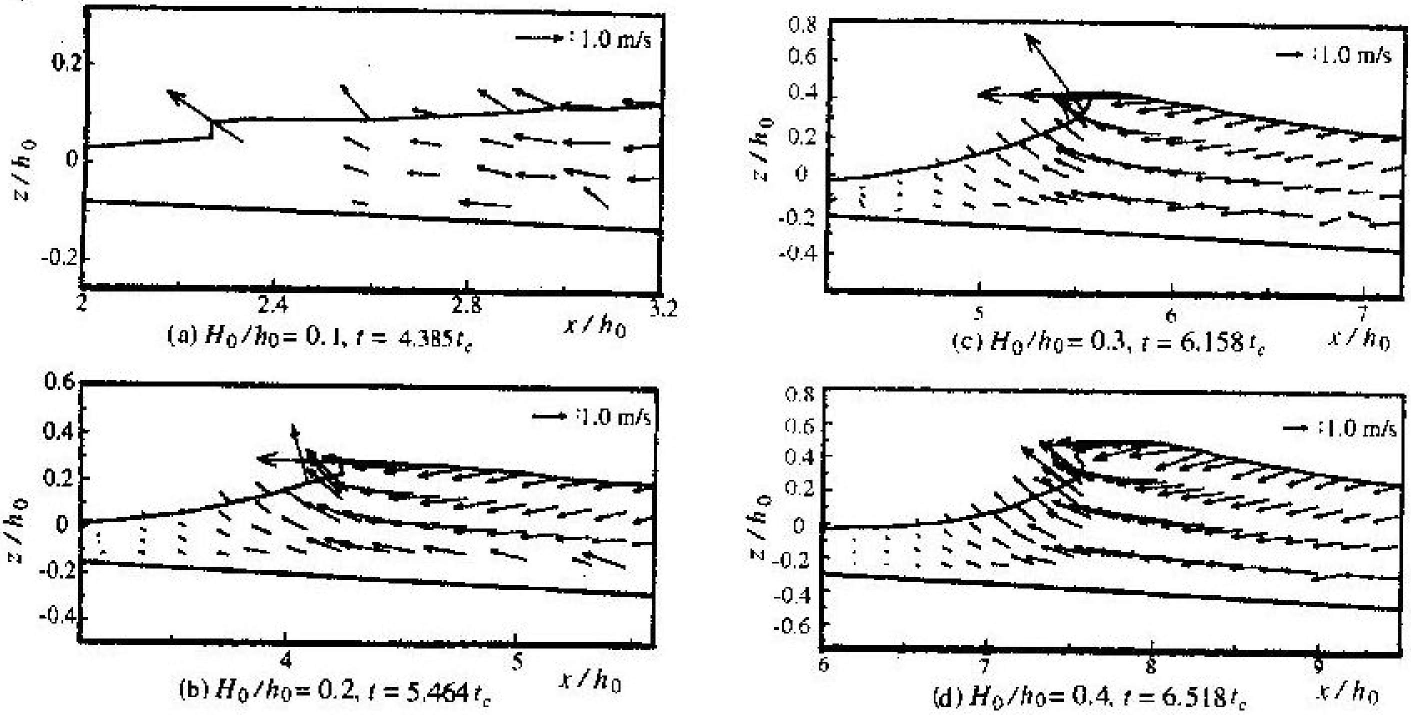

Fig. 9. Distribution of fluid velocities at breaking on slope $s = 1:20$.

Synolakis, C. E., 1987. The Runup of Solitary Waves, *J. Fluid Mech.*, **185**, 523 ~ 545.

Yamada, H. *et al.*, 1968. On the Highest Wave in Water, *Bull. Disas*, Prev. Res. Inst. Kyoto Univ., **18**(part 2), No. 135, 1 ~ 23.