

# A Study of the Wave Climate of Northern Taiwan

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## ABSTRACT

Possible long-term wave height distributions for the northern part of Taiwan are studied. Several statistical models for the extreme value distribution are used for the study. It is found that the lognormal, the Weibull, the gamma, the Fisher-Tippett Type I, and the Generalized Extreme Value (GEV) distributions can be used. Furthermore, it is found that among the six models considered, the lognormal and the GEV models can fit the empirical distribution more satisfactorily.

**KEY WORDS:** significant wave heights; long-term distributions; generalized extreme value distribution.

## INTRODUCTION

Among all the forces to be considered in designing coastal structures, those due to waves are probably the most important. Since these constructions are often designed to endure environmental impacts for a relatively long time, say 50, 100 years or more, it is vital that an estimate of the possible occurrences can be made. However, as is common for most nature phenomena, long-term records are either rather scarce, or simply does not exist. A way out of this predicament is to use statistical models for the extremes.

It should be noted that, there is absolutely no theoretical justification that any of the available models should be used. This has been pointed out by many researchers in the past. One way out of this dilemma is to choose the model that has the best fit of the measured results. Among all the possible candidates, the lognormal, the exponential, the gamma, the Weibull, and the Fisher-Tippett Type I (Gumbel) distributions are most frequently adopted.

Bretschneider & Rocheleau (1978) studied the wave climate of Keahole Point, Hawaii. They used the Gaussian, the lognormal, the Weibull, and the Fisher-Tippett Type I (Gumbel) distributions as possible models. The procedures of determining design wave heights were reviewed by Isaacson & Mackensie (1981) in some detail. They examined the performances of the lognormal and the three types of the Extreme Value distributions in modelling long-term wave statistics. In a series

of papers Ochi (1978; see also Ochi et al., 1986; Ochi, 1992) considered the possibility of fitting measured wave heights with the lognormal, the Extreme Value Type III, and the generalized gamma distribution. It is concluded that the generalized gamma distribution is the most suitable one to be used in estimating extreme wave heights (Ochi, 1992).

Quite often, the combination of two extreme value distribution functions is also used. Haver (1985), for example, examined the wave climate of Tromøflaket, Norway. Records from the year 1977 to 1981 were used. A combination of the lognormal and the Weibull distribution function was used by him. It is concluded that, for wave heights smaller than a specific value the lognormal distribution can be used, whereas the Weibull distribution should be used for wave heights larger than this specific value. Similar conclusions were also reached by Shi (1991), i.e., the lognormal distribution is a better choice for fitting data of lower sea states.

Bobée & Ashkar (1991) have pointed out that, to be able to describe the statistical properties of a nature occurrence a distribution function should have at least two parameters or more. On the other hand, it is not necessarily true that distribution functions having more than three parameters will have a better performance. The chances of estimating the values of the parameters wrongly will grow with increasing number of parameters due to sampling uncertainties.

This is demonstrated by Andrew & Hemsley (1990). They used a bootstrap resampling approach to explore the performances of the Gumbel, and the three-parameter Weibull distributions. Predicted extreme wave heights were compared with measured and hindcast data. It was found that, although the latter is more flexible due to its three adjustable parameters, nevertheless the former is more adequate in predicting extreme events. The three-parameter Weibull distribution was also used by Naffaa (1995) to fit measured significant wave heights from two stations along the Nile Delta coast. However, after studying wave data from 19 measuring stations along the coast of China, Yao et al. (1993) found that, among all the models considered by them, the results of the four-parameter exponential-gamma distribution are most satisfactory. Longtime waveheight- and period

distributions were also studied by Teng (1998, see also Teng & Palao, 1996). 17 years of hourly wave data from three buoy stations in the Gulf of Mexico were used. Besides the lognormal and the Weibull distributions, the authors also used a modified version of the lognormal distribution to fit the data. The latter model was found to fit the data more satisfactory than the other two models.

As mentioned earlier, when compared with the intended lifetime of the construction, quite often the available wave records are rather short. It is not exactly known how many years of data are necessary such that reliable extrapolations can be made. Berger (1993), for example, pointed out that, one should have at least a wave record of three-year length to be able to carry out extremal analysis. He used the Fisher-Tippett Type I distribution as his model equation. Naess (1998), on the other hand, have found that in order to extrapolate to long return period one should have wave records of about 20 years or more. This is a rather requirement stringent, because it cannot be fulfilled most of the time. Furthermore, the author pointed out that, the Weibull distribution is not an appropriate model for the asymptotic extreme wave height distribution.

In this paper, we present our results of analyzing the records of wave height measured in the northern part of Taiwan. Besides the models commonly used by other researchers, we also used the Generalized Extreme Value distribution as a model for the long-term wave heights. In the following, we further divide our paper into four parts. In Section 2 we briefly describe the climatological conditions of Taiwan; as well as the wave measuring stations where the data were acquired. Short descriptions of the models will be given in Section 3, while in Section 4 we present our results. A short conclusion then closes this paper.

## THE WAVE DATA

Taiwan is an island which lies at the southeastern coast of Mainland China, with its eastern coast facing the Pacific Ocean. Relatively strong northeast wind blowing often in winter seasons; whereas mild southwest winds are common in the summer. On the average, it has to face two typhoon invasions per year, which, when it happens, occurs in summer and autumn seasons.



Figure 1. Schema of the wave measuring stations.

Wave data from 6 measuring stations, namely the Pi-Tou-Chiao (in the following we will use the abbreviation PTC), the Long-Tong (LT), the Hsin-Chu (HC), the CBK-11, the Long-Men (LM), and the Keelung harbour (KL), were used for the present analysis. The first four measuring stations are maintained by the Central Weather Bureau (CWB). The data from LM is kindly provided by a consulting company,

and the Keelung Harbour Bureau is responsible for the last measuring station. Figure 1 shows the approximate locations of these measuring stations.

Not all records of the measuring stations are of equal length. Table 1 lists the records available for each measuring station. It can be seen from Table 1 that LM has the shortest, whereas PTC has the longest wave data. It was mentioned earlier that Berger (1993) has pointed out that at least 3 year of wave data are needed for extremal analysis. In this respect, it is not sure whether the records of LM can be used. We will, nevertheless, continue our analysis in this paper.

Table 1. Information concerning the measuring sites. NA = not available. For data types: All =  $H_s$ ,  $H_{1/10}$ , Hmean, and Hmax with corresponding periods; Hs only = only significant wave heights and periods are available.

| Measuring station    | Water depth [m] | Record length [min/hr] | Location                              | Recording period         | Data type  |
|----------------------|-----------------|------------------------|---------------------------------------|--------------------------|------------|
| PiTouChiao 1 (PTC 1) | 55              | 20/2                   | 25° 08' 09" N<br>121° 55' 31" E       | Oct. 1980 –<br>Jul. 1988 | All        |
| PiTouChiao 2 (PTC 2) | 55              | 10/1                   | 25° 08' 09" N<br>121° 55' 31" E       | Aug. 1991 –<br>Oct. 1999 | All        |
| PiTouChiao 3 (PTC 3) | 55              | 10/1                   | 25° 08' 09" N<br>121° 55' 31" E       | July 1998 –<br>Dec. 2000 | All        |
| CKB 11               | NA              | 10/1                   | 24° 47' 07" N<br>120° 39' 53" E       | Oct. 1997 –<br>Sep. 2000 | All        |
| LongTong (LT)        | 22              | 10/1                   | 25° 05' 42" N<br>121° 55' 28" E       | Oct. 1998 –<br>Dec. 2000 | Hs<br>only |
| HsinChu (HC)         | 22              | 10/1                   | 24° 54' 51" N<br>120° 56' 31" E       | May 1997 –<br>Dec. 2000  | Hs<br>only |
| LongMen (LM)         | 14              | 10/1                   | NA                                    | Apr. 2002 –<br>July 2003 | All        |
| Keelung (KL)         | 31.60           | Daily                  | 25° 09' 54.35" N<br>121° 44' 57.94" E | Jan. 1983 –<br>Dec. 1990 | All        |

## THE STATISTICAL MODELS

A total of 12 statistical models were used for the study. Among these 12, only six, i.e., the Weibull, the 2- & 3-parameter lognormal, the gamma, the Fisher-Tippett Type I, and the Generalized Extreme Value distribution are found to always fit the data. We therefore concentrate ourselves in the following only on these models. These are:

a. The Weibull distribution

$$f(x'; \alpha_w, \beta_w) = \alpha_w \beta_w x'^{\alpha_w - 1} \exp(-\beta_w x'^{\alpha_w}) \quad \alpha_w, \beta_w > 0 \quad (1)$$

b. The two-parameter lognormal distribution

$$p(x'; \sigma_y, \mu_y) = \frac{1}{x' \sigma_y \sqrt{2\pi}} \exp\left\{-\frac{[\ln(x') - \mu_y]^2}{2\sigma_y^2}\right\} \quad \text{where } y = \ln(x'), \quad (2)$$

c. The three-parameter lognormal distribution

$$p(x'; \sigma_y, \mu_y, \gamma) = \frac{1}{(x' - \gamma)\sigma_y \sqrt{2\pi}} \exp\left\{-\frac{[\ln(x' - \gamma) - \mu_y]^2}{2\sigma_y^2}\right\} \quad (3)$$

d. The gamma distribution

$$p(x'; \eta, \zeta) = \frac{\zeta^\eta}{\Gamma(\eta)} x'^{\eta-1} e^{-\zeta x'} \quad \text{where } \Gamma(\eta) = \int_0^\infty x'^{\eta-1} e^{-x'} dx' \quad (4)$$

e. The Fisher-Tippett Type I distribution

$$p(x') = \alpha_G \exp\{-\alpha_G(x' - \beta_G) - \exp[x' - \beta_G]\} \quad \alpha_G > 0 \quad (5)$$

f. The Generalized Extreme Value distribution

$$f(x) = \frac{1}{\alpha} \left(1 - k \frac{x-u}{\alpha}\right)^{\frac{1}{k}-1} \exp\left[-\left(1 - k \frac{x-u}{\alpha}\right)^{\frac{1}{k}}\right] \quad (6)$$

Details concerning the first five distribution models can be found in textbooks concerning statistics (See, for example, Hahn & Shapiro, 1967; Haan, 1991). The properties of the Generalized Extreme Value (GEV) distribution are described in detail by Rao & Hamed (2000). These will be omitted here for brevity.

## RESULTS AND DISCUSSION

Figure 2 shows the interrelationship between  $H_{1/10}$  and the  $H_{1/3}$  for the measuring station PTC. The waves were measured from October 1980 to July 1988. A regression line is also drawn in the Figure. It can be seen that the empirical relationships between the significant and the one-tenth wave heights is very close to the expression given by Goda (2000):

$$H_{1/10} = 1.27H_{1/3} = 2.03H_{\text{mean}} \quad (7)$$

Table 2 summarizes the calculated relations between various wave heights.

Table 2. Relationships between various wave heights for the measuring stations PTC, CKB-11 and KL.

| Station Relations                | PTC 1 | PTC 2 | PTC 3 | CKB-11 | Keelung | Rayleigh | Empirical |
|----------------------------------|-------|-------|-------|--------|---------|----------|-----------|
| $H_{1/10}$ vs. $H_{1/3}$         | 1.251 | 1.249 | 1.235 | 1.242  | 1.177   | 1.27     | 1.15~1.45 |
| $H_{1/10}$ vs. $H_{\text{mean}}$ | 1.590 | 1.982 | 1.851 | 1.906  | 1.792   | 2.03     | --        |
| $H_{1/3}$ vs. $H_{\text{mean}}$  | 1.274 | 1.590 | 1.501 | 1.539  | 1.006   | 1.60     | 1.40~1.75 |
| $H_{\text{max}}$ vs. $H_{1/3}$   | 1.627 | 1.518 | 1.530 | 1.473  | 1.512   | --       | 1.80~2.70 |

Figure 3 shows the probability distribution of maximum wave heights measured at PTC for the periods Aug. 1991 to Dec. 2000. The curves of the six models are also shown in the figure for comparison. It can be seen from the histogram that the distribution is skewed to the right, and only the two-parameter lognormal and the GEV distribution functions can match the peak of the histogram.

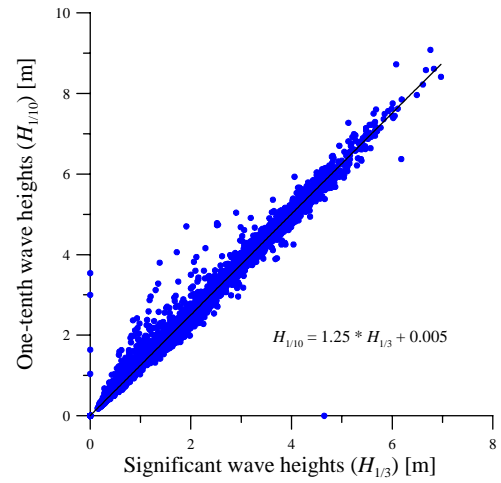


Figure 2. Interrelationship between  $H_{1/10}$  and  $H_{1/3}$  for PTC (1980-1988)

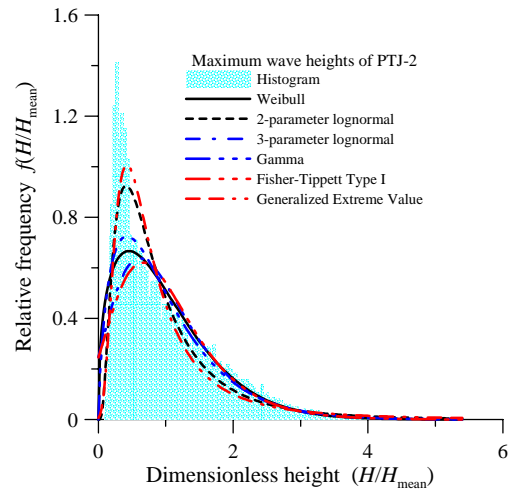


Figure 3. Probability distribution of maximum wave heights for the measuring station PTC (Aug. 1991 - Oct. 2000).

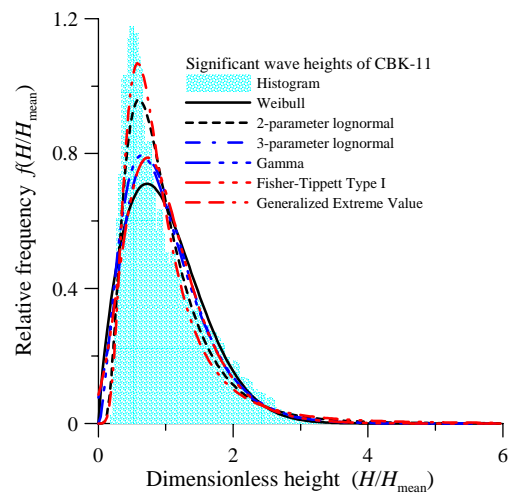


Figure 4. Probability distribution of significant wave heights for CKB-11.

As another example, we show the probability distribution of the significant wave heights measured at the CBK-11 station. From Figure 4, it can be seen that, similar as Figure 3, the two distribution functions mentioned above can fit the peak of the histogram rather well. This, however, is at the expense of less well match of the values higher than the mean value of the maximum wave heights.

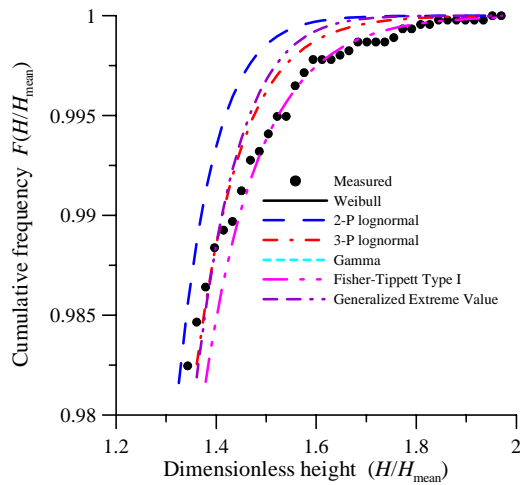


Figure 5. Cumulative probability distribution of significant wave heights for the measuring station LD.

Cumulative probability distributions of significant wave heights for the measuring station LD are shown in Figure 5. Notice how well the curve of the Gumbel distribution follows the measured data. This is not the case for the maximum wave heights measured in LM, as shown in Figure 6. In this figure, the curves of the three-parameter lognormal and the generalized extreme value distributions follow roughly the empirical distribution. On the other hand, the curves of the Weibull and Gumbel distributions underestimate, and those of the gamma and two-parameter lognormal distributions overestimate.

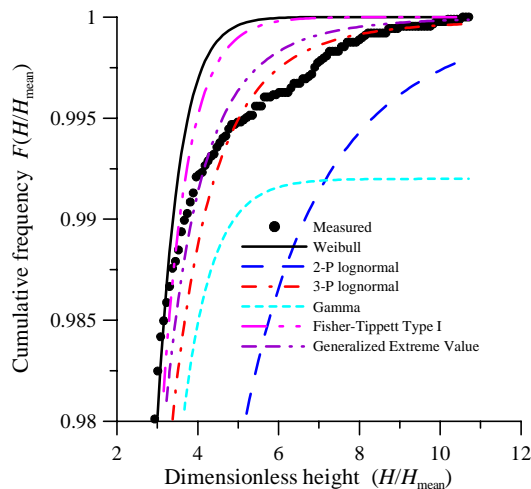


Figure 6. Cumulative probability distribution of the maximum wave heights for the measuring station LM

## CONCLUSION

Wave heights from six measuring stations in the northern part of Taiwan were analyzed here in this paper. Six model equations, usually used in the extremal distribution, are used to fit the empirical data. It was found that:

- The interrelationships between various wave heights are all larger than the theoretical values based on the theoretical Rayleigh distribution. However, these match those found in the literature. The reason for the deviations from the Rayleigh distribution is not clear at present.
- Only the curves of the two- or three-parameter lognormal, and the generalized extreme value distributions can fit the empirical peak of measured data. All other distribution functions have difficulties following the trend. It is, however, not clear at present, whether this would affect the value of predicted return wave height.
- It can be seen in Figure 6 that the gamma and the two-parameter lognormal distribution overestimate the wave heights. However, judging from Figure 5, it is conjectured that this could also be due to sampling variability associated with the data from LM. As can be seen from Table 1, the data has a length of a little more than a year.

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