非定常熱擴散溫度-渦度-流函數法

靜止流體其溫度一樣，以 $T_{\min}$ 表示，流體溫度成 T 時，密度 ρ 亦產生變化。溫度上昇時流體膨漲密度減少而產生浮力，溫度下降時密度增加而下沉，並產生流。考量密度變化的質量守恆方程式如下

$$\frac{\partial \rho}{\partial t}+\frac{\partial\left(\rho v_1\right)}{\partial x_1}+\frac{\partial\left(\rho v_2\right)}{\partial x_2}=0$$

不考量密度變化的質量守恆方程式如下

$$\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}=0$$

流體流速分量應滿足下列運動方程式

$$\rho\left(\frac{\partial v_1}{\partial t}+v_1\frac{\partial v_1}{\partial x_1}+v_2\frac{\partial v_1}{\partial x_2}\right)=\frac{\partial \sigma_{11}}{\partial x_1}+\frac{\partial \sigma_{12}}{\partial x_2}+b_1$$

$$\rho\left(\frac{\partial v_2}{\partial t}+v_1\frac{\partial v_2}{\partial x_1}+v_2\frac{\partial v_2}{\partial x_2}\right)=\frac{\partial \sigma_{21}}{\partial x_1}+\frac{\partial \sigma_{22}}{\partial x_2}+b_2$$

$$\sigma_{ij}=-p\delta_{ij}+\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)$$

在熱流體問題，因溫度效應引起的物體力為

$$b_1 = 0$$

$$b_2 = \rho g \beta \left(T - T_{\min}\right),\quad \beta = \text{體膨脹率}$$

解析熱流體問題，除連續方程式、運動方程式外必須考量下列能量方程式。

$$\frac{\partial T}{\partial t}+v_1\frac{\partial T}{\partial x_1}+v_2\frac{\partial T}{\partial x_2}-\frac{k}{\rho c}\left(\frac{\partial^2 T}{\partial {x_1}^2}+\frac{\partial^2 T}{\partial {x_2}^2}\right)=0$$

將上式無因次化得

$$Pe\left(\dot{\theta}+u_j\theta_{,j}\right)-\theta_{,jj}=0$$

$$Pe=LU/\left(k/\rho c\right)$$

L = 代表長　U = 基準流速

k = 熱傳導率　ρ = 密度　c = 比熱

$$u_j=v_j/U$$

$$\theta=\left(T-T_{\min}\right)/\left(T_{\max}-T_{Min}\right)\quad T=溫度$$

若使用渦度 ω 及流函數 ψ ，連續方程式及運動方程式可變形成下列渦度輸送方程式

$$\mathrm{Re}\left(\dot{\omega}+u_j\omega_{,j}\right)-\omega_{,jj}+f_{2,1}=0 \qquad (1)$$

渦度為

$$\omega=-\psi_{,jj}$$

$$\mathrm{Re}=LU/\left(\mu/\rho\right)$$

$$f_2=\frac{Gr}{\mathrm{Re}}\theta$$

$$Gr=\frac{\rho g\beta\left(T-T_{\min}\right)L^3}{\left(\mu/\rho\right)^2}$$

將(1)式，如非定常移流擴散使用含時間項的基本解所示方法，乘基本解後積分可得能量方程式的積分方程式如下

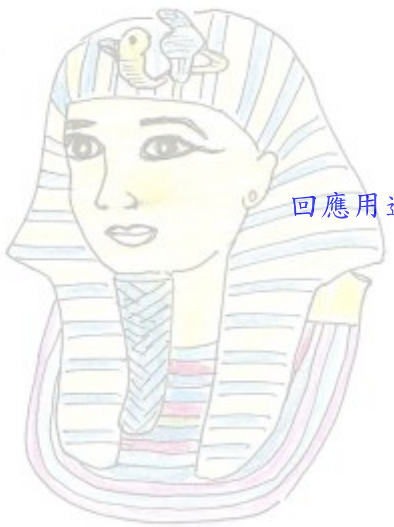
$$\gamma\theta\left(r=0,\tau_2\right)Pe+\int_\Gamma\int_{\tau_1}^{\tau_2}\theta w_{,n}d\tau d\Gamma=$$
$$\int_\Gamma\int_{\tau_1}^{\tau_2}\theta_{,n}d\tau d\Gamma-\int_\Omega\int_{\tau_1}^{\tau_2}Peu_j\theta_{,j}wd\tau d\Omega+\int_\Omega\left(wPe\theta\right)_{\tau=\tau_1}d\Omega$$

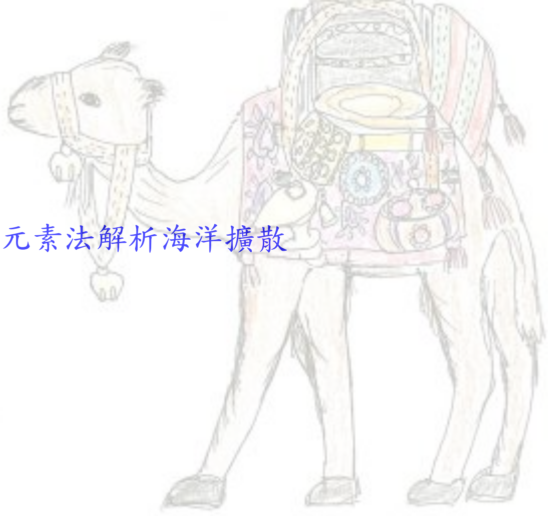
渦度輸送方程式的積分方程式為

$$\gamma\omega\left(r=0,\tau_2\right)Re+\int_\Gamma\int_{\tau_1}^{\tau_2}w_{,n}d\tau d\Gamma=$$
$$\int_\Gamma\int_{\tau_1}^{\tau_2}\omega_{,n}wd\tau d\Gamma-\int_\Omega\int_{\tau_1}^{\tau_2}Reu_j\omega_{,j}wd\tau d\Omega+\int_\Omega\left(wPe\omega\right)_{\tau=\tau_1}d\Omega-\int_\Gamma\int_{\tau_1}^{\tau_2}f_{2,1}wd\tau d\Omega$$

渦度積分方程式如下

$$\gamma\psi + \int_{\Gamma} \psi w_{,n} d\Gamma = \int_{\Gamma} \psi_{,n} w d\tau d\Gamma - \int_{\Omega} \omega w d\Omega$$

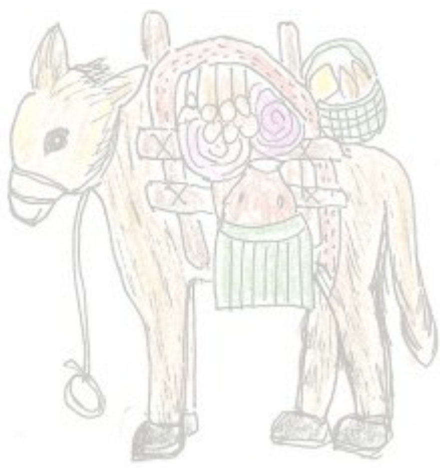




回應用邊界元素法解析海洋擴散

載滿珠寶的駱駝

2011 埃及尼羅河之旅



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